Journal of Engineering Physics and Thermophysics, Vol. 77, No. 1, 2004

USE OF ASYMPTOTIC METHODS IN THE TEMPERATURE PROBLEM ON FILTRATION OF GASSED OIL IN A POOL

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UDC 532.546

The temperature fields arising in the filtration of a gassed liquid in saturated porous media have been theoretically investigated with simultaneous consideration for the gas-phase imperfection, phase transitions, and heat exchange between an oil pool and the surrounding rocks.

Data of thermal investigations of oil wells are widely used in the oil-gas industry for revealing the yielding intervals and the intervals of gas transfer and gas inflow behind a drill pipe string and estimating the technical state of wells [1]. In this case, it is important to take into account temperature changes arising in the process of movement of liquids and gases in porous media due to their imperfection and the adiabatic effects. By now the theory of stationary temperature fields arising in a stationary pressure field has been developed for liquids and gases and in nonstationary pressure fields — for liquids. The problem of filtration of a gassed oil is one of the most important in the theory of thermometry. The importance of its solution, apparently, was substantiated for the first time in [2]. Investigations of the temperature fields arising in the filtration of a gassed liquid were carried out in [3, 4] with the use of the equation of an ideal-gas state without regard for the heat transfer to the surrounding rocks, but they did not properly explain this phenomenon since the Joule–Thomson coefficient is equal to zero for an ideal gas.

The aim of the present work is analytical solution of temperature problems on filtration of a gassed liquid with allowance for the heat exchange with the surrounding rocks, phase transitions, and gas-phase imperfection and numerical calculations based on the solutions obtained. This called for the complex use of mathematical-physics methods: integral transforms, the method of characteristics, and asymptotic methods [5].

Formulation of the Complete Problem and Expansion in Terms of a Parameter. A temperature problem is considered in a cylindrical coordinate system where a porous pool is surrounded by two semi-infinite regions with plane interfaces perpendicular to the z axis ($z = \pm h$, see Fig. 1). The first and second regions are impenetrable, and the central region of thickness $2h$ is positioned horizontally and is saturated with a gassed liquid. The case of radial movement of a gassed oil in the central region $-h < z < h$ from infinity to the well of zero radius is investigated. It is assumed that the saturation pressure is lower than the pressure in the pool but higher than the pressure in the well, $P_w < P_s < P_c$; therefore, a two-zone flow is realized in the pool. We will consider the temperature problem assuming that the temperatures of the oil, gas, and skeleton of the porous medium are equal at each point [6] and only the radial coordinate of the convective-heat-transfer rate differs from zero, i.e., $U_r \neq 0$, $U_\varphi = 0$, and $U_z = 0$. The mathematical formulation of the problem for the first and second regions is represented by the heat-conduction equation and for the central region by the barothermal-effect equation [7]. We will formulate the problem in dimensionless variables:

$$\frac{\partial T_1}{\partial t} - \frac{a_{r1}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right) - \frac{\partial^2 T_1}{\partial z^2} = 0\,, \quad z > 1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{1}$$

$$\frac{\partial T_2}{\partial t} - \frac{a_{r2}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right) - \frac{a_{z2}}{a_{z1}}\frac{\partial^2 T_2}{\partial z^2} = 0\,, \quad z < -1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{2}$$

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1062-0125/04/7701-0062

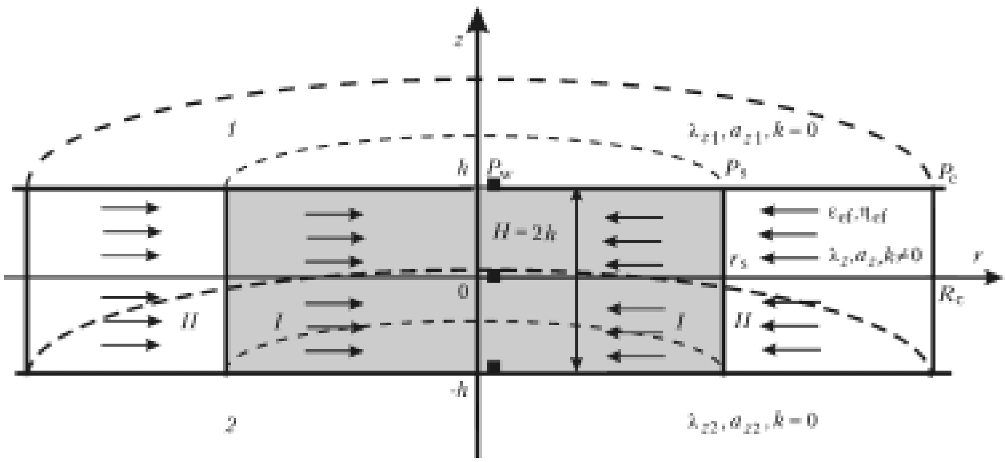

Fig. 1. Geometry of the problem: 1, 2) impenetrable regions; I) zone of two-phase oil and free gas movement; II) zone of oil movement.

$$\frac{\partial T}{\partial t} + U_{\text{ef}}(r,t)\left(\frac{\partial T}{\partial r} + \varepsilon_{\text{ef}} + \frac{\partial P}{\partial r}\right) - \eta_{\text{ef}}\,\Pi\,\frac{\partial P}{\partial t} + q_1 = \frac{a_r}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\chi}{\varepsilon}\frac{\partial^2 T}{\partial z^2}\,, \quad |z| < 1\,, \quad r \ge 0\,, \quad t > 0\,; \tag{3}$$

The conditions of equality of temperatures and heat flows are set at the interfaces

$$T\big|_{z=1} = T_1\big|_{z=1}\,, \quad \varepsilon\frac{\partial T_1}{\partial z}\bigg|_{z=1} = \frac{\partial T}{\partial z}\bigg|_{z=1}\,; \tag{4}$$

$$T\big|_{z=-1} = T_2\big|_{z=-1}\,, \quad \varepsilon\frac{\partial T_2}{\partial z}\bigg|_{z=-1} = \frac{\lambda_{z1}}{\lambda_{z2}}\frac{\partial T}{\partial z}\bigg|_{z=-1}\,; \tag{5}$$

temperature disturbances are absent at the initial instant of time

$$T\big|_{t=0} = T_1\big|_{t=0} = T_2\big|_{t=0} = 0\,; \tag{6}$$

the boundary condition has the form

$$\lim T_i\big|_{r+|z|\to\infty} = 0\,, \tag{7}$$

$\chi = c_1\rho_1/(c\rho)$, $z = z_d/h$, $t = t_d a_{z1}/h^2$, $T = T_d/T_0$, $U_{\text{ef}} = U_{\text{ef,d}}h/a_{z1}$, $r = r_d/h$, $P = P_d/P_0$, $q_1 = Lq_d h^2/(T_0\lambda_{z1})$, $\varepsilon_{\text{ef}} = \varepsilon_{\text{ef,d}}P_0/T_0$, and $\eta_{\text{ef}} = \eta_{\text{ef,d}}P_0/T_0$. $T_0 = \varepsilon_{\text{ef}}P_0$ can be used as T_0.

It is assumed that the solution is bounded at all points $r > 0$ and, at $r = 0$, the derivative of the temperature with respect to the radial coordinate is equal to zero in the corresponding regions. The lower indices 1 and 2 are related to the parameters of the first and second media, respectively. The density function of the free-gas sources q, the rate of convective heat transfer U_{ef}, and the other parameters are determined from the hydrodynamic problem. An important point is that temperature-independent parameters are used in constructing asymptotic approximations.

It is very difficult to solve the problem analytically. To obtain approximate solutions, we used an asymptotic method. The heat conductivity λ_{z1} is conductive, and λ_z includes an additional convective heat conductivity. This makes it possible to take $\varepsilon = \lambda_{z1}/\lambda_z$ as an expansion parameter. Note that only if the expansion parameter is chosen in this way is the problem reduced, in the zero approximation in a particular case, to the known "lumped-capacitance scheme" [8–11]. We will find a solution of the problem in the form of asymptotic series in terms of ε:

$$T = T^{(0)} + \varepsilon T^{(1)} + \varepsilon^2 T^{(2)} + \ldots + \varepsilon^n T^{(n)} + \ldots , \tag{8}$$

$$T_1 = T_1^{(0)} + \varepsilon T_1^{(1)} + \varepsilon^2 T_1^{(2)} + \ldots + \varepsilon^n T_1^{(n)} + \ldots , \quad T_2 = T_2^{(0)} + \varepsilon T_2^{(1)} + \varepsilon^2 T_2^{(2)} + \ldots + \varepsilon^n T_2^{(n)} + \ldots . \tag{9}$$

where the lower indices of T are related to the number of the region and the upper indices correspond to the serial number of the approximation. Substituting (8) and (9) into (1)–(7) and grouping the terms of the same order with respect to ε, we obtain

$$\left(\frac{\partial T_1^{(0)}}{\partial t} - \frac{a_{r1}}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1^{(0)}}{\partial r} \right) - \frac{\partial^2 T_1^{(0)}}{\partial z^2} \right) + \varepsilon \left(\frac{\partial T_1^{(1)}}{\partial t} - \frac{a_{r1}}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1^{(1)}}{\partial r} \right) - \frac{\partial^2 T_1^{(1)}}{\partial z^2} \right) + \ldots = 0 , \tag{10}$$

$$\left(\frac{\partial T_2^{(0)}}{\partial t} - \frac{a_{r2}}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T_2^{(0)}}{\partial r} \right) - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(0)}}{\partial z^2} \right) + \varepsilon \left(\frac{\partial T_2^{(1)}}{\partial t} - \frac{a_{r2}}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T_2^{(1)}}{\partial r} \right) - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(1)}}{\partial z^2} \right) = 0 , \tag{11}$$

$$- \chi \frac{\partial^2 T^{(0)}}{\partial z^2} + \varepsilon \left[\frac{\partial T^{(0)}}{\partial t} + U_{\text{ef}}(r, t) \left(\frac{\partial T^{(0)}}{\partial r} + \varepsilon_{\text{ef}} \frac{\partial P}{\partial r} \right) - \eta_{\text{ef}} \Pi \frac{\partial P}{\partial t} + q_1 - \frac{a_r}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(0)}}{\partial r} \right) - \right.$$
$$\left. - \chi \frac{\partial^2 T^{(1)}}{\partial z^2} \right] + \varepsilon^2 \left[\frac{\partial T^{(1)}}{\partial t} + U_{\text{ef}}(r, t) \frac{\partial T^{(1)}}{\partial r} - \frac{a_r}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(1)}}{\partial r} \right) - \chi \frac{\partial^2 T^{(2)}}{\partial z^2} \right] + \ldots = 0 , \tag{12}$$

$$\left(T^{(0)} \big|_{z=1} - T_1^{(0)} \big|_{z=1} \right) + \varepsilon \left(T^{(1)} \big|_{z=1} - T_1^{(1)} \big|_{z=1} \right) + \varepsilon^2 \left(T^{(2)} \big|_{z=1} - T_1^{(2)} \big|_{z=1} \right) + \ldots = 0 , \tag{13}$$

$$\left(T^{(0)} \big|_{z=-1} - T_2^{(0)} \big|_{z=-1} \right) + \varepsilon \left(T^{(1)} \big|_{z=-1} - T_2^{(1)} \big|_{z=-1} \right) + \varepsilon^2 \left(T^{(2)} \big|_{z=-1} - T_2^{(2)} \big|_{z=-1} \right) + \ldots = 0 , \tag{14}$$

$$\left. \frac{\partial T^{(0)}}{\partial z} \right|_{z=1} + \varepsilon \left(\left. \frac{\partial T^{(1)}}{\partial z} \right|_{z=1} - \left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} \right) + \varepsilon^2 \left(\left. \frac{\partial T^{(2)}}{\partial z} \right|_{z=1} - \left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} \right) + \ldots = 0 , \tag{15}$$

$$\left. \frac{\lambda_{z1}}{\lambda_{z2}} \frac{\partial T^{(0)}}{\partial z} \right|_{z=-1} + \varepsilon \left(\left. \frac{\lambda_{z1}}{\lambda_{z2}} \frac{\partial T^{(1)}}{\partial z} \right|_{z=-1} - \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right) + \varepsilon^2 \left(\left. \frac{\lambda_{z1}}{\lambda_{z2}} \frac{\partial T^{(2)}}{\partial z} \right|_{z=-1} - \left. \frac{\partial T_2^{(1)}}{\partial z} \right|_{z=-1} \right) + \ldots = 0 , \tag{16}$$

$$T^{(i)} \big|_{t=0} = 0 , \quad T_1^{(i)} \big|_{t=0} = 0 , \quad T_2^{(i)} \big|_{t=0} = 0 , \quad i = 0, 1, 2, \ldots , \tag{17}$$

$$\lim T^{(i)} \big|_{r+|z| \to \infty} = 0 , \quad \lim T_1^{(i)} \big|_{r+|z| \to \infty} = 0 , \quad \lim T_2^{(i)} \big|_{r+|z| \to \infty} = 0 , \quad i = 0, 1, 2, \ldots \tag{18}$$

For the zero approximation, from (12) we have $\partial^2 T^{(0)} / \partial z^2 = 0$. From this it follows herefrom that the derivative of the zero approximation with respect to the z coordinate is independent of z, i.e., $\partial T^{(0)} / \partial z = \text{const}$. Since, for the zero approximation, it follows from (15) and (16) that $\left.\frac{\partial T^{(0)}}{\partial z}\right|_{z=\pm 1} = 0$, we obtain $\partial T^{(0)}/\partial z = 0$. The latter means that $T^{(0)}$ is independent of z and is a function of only r and t, i.e., $T^{(0)} = T^{(0)}(r, t)$. Equating the expression before the cofactor of the first order with respect to ε to zero in (12), we obtain the following equation:

$$\frac{\partial^2 T^{(1)}}{\partial z^2} = \frac{1}{\chi}\left[\frac{\partial T^{(0)}}{\partial t} + U_{\text{ef}}(r, t)\left(\frac{\partial T^{(0)}}{\partial r} + \varepsilon_{\text{ef}}\frac{\partial P}{\partial r}\right) - \eta_{\text{ef}}\,\Pi\,\frac{\partial P}{\partial t} + q_1 - \frac{a_r}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(0)}}{\partial r}\right)\right] = A(r, t)\,. \tag{19}$$

Equation (19) is "linked" in the sense that it involves expansion coefficients of the zero and the first order ($T^{(0)}$ and $T^{(1)}$), which makes the solution of the problem difficult. Below are given transforms that allowed us to "decompose" Eq. (19) by elimination of $T^{(1)}$ from it. Since $T^{(0)}$ and P are independent of z, $A(r, t)$ is also independent of z according to (19). Integrating (19) twice, we obtain an expression for the temperature in the pool in the first approximation:

$$\frac{\partial T^{(1)}}{\partial z} = zA(r, t) + B(r, t)\,, \quad T^{(1)} = \frac{z^2}{2}A(r, t) + zB(r, t) + E(r, t)\,, \tag{20}$$

where $B(r, t)$ and $E(r, t)$ are also independent of z.

Equating the expressions before the cofactor of the first order with respect to ε in (15) and (16), we will have

$$\left.\frac{\partial T^{(1)}}{\partial z}\right|_{z=1} = \left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1}, \quad \left.\frac{\partial T^{(1)}}{\partial z}\right|_{z=-1} = \left.\frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial T_2^{(0)}}{\partial z}\right|_{z=-1}. \tag{21}$$

Let us write expression (20) for the temperature gradient $\partial T/\partial z$ at $z = 1$ and $z = -1$ and, using equalities (21), construct a system of equations for the unknown coefficients $A(r, t)$ and $B(r, t)$:

$$\left.\frac{\partial T^{(1)}}{\partial z}\right|_{z=1} = A(r, t) + B(r, t) = \left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1}; \tag{22}$$

$$\left.\frac{\partial T^{(1)}}{\partial z}\right|_{z=-1} = -A(r, t) + B(r, t) = \left.\frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial T_2^{(0)}}{\partial z}\right|_{z=-1}. \tag{23}$$

Solving the system of equations (22)–(23) for $A(r, t)$ and $B(r, t)$, we obtain

$$A(r, t) = \frac{1}{2}\left(\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1} - \left.\frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial T_2^{(0)}}{\partial z}\right|_{z=-1}\right), \quad B(r, t) = \frac{1}{2}\left(\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1} + \left.\frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial T_2^{(0)}}{\partial z}\right|_{z=-1}\right). \tag{24}$$

Substitution of expression (24) for the quantity $A(r, t)$ into (19) gives an equation involving expansion coefficients of the zero order only. Using it and the conditions of equality of the first terms in (10) and (11) to zero, we write the final formulation of the problem for the zero approximation:

$$\frac{\partial T_1^{(0)}}{\partial t} - \frac{a_{r1}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1^{(0)}}{\partial r}\right) - \frac{\partial^2 T_1^{(0)}}{\partial z^2} = 0\,, \quad z > 1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{25}$$

$$\frac{\partial T_2^{(0)}}{\partial t} - \frac{a_{r2}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2^{(0)}}{\partial r}\right) - \frac{a_{z2}}{a_{z1}}\frac{\partial^2 T_2^{(0)}}{\partial z^2} = 0\,, \quad z < -1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{26}$$

$$\frac{\partial T^{(0)}}{\partial t} + U_{\text{ef}}\,(r,\,t)\,\frac{\partial T^{(0)}}{\partial r} - \frac{a_r}{a_{z1} r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T^{(0)}}{\partial r} \right) - \frac{\chi}{2}\left(\left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right) =$$

$$= -\,\varepsilon_{\text{ef}} U_{\text{ef}}\,\frac{\partial P}{\partial r} + \eta_{\text{ef}}\,\ddot{\text{I}}\,\frac{\partial P}{\partial t} - q_1 = f\,(r,\,t)\,, \quad |\,z\,| < 1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{27}$$

$$T_1^{(0)}\,|_{z=1} = T_2^{(0)}\,|_{z=-1} = T^{(0)}\,; \tag{28}$$

$$T^{(0)}\,|_{t=0} = T_1^{(0)}\,|_{t=0} = T_2^{(0)}\,|_{t=0} = 0\,; \tag{29}$$

$$\lim T_i^{(0)}\,|_{r+|z| \to \infty} = 0\,. \tag{30}$$

Equations (25)–(27) are interrelated since (27), along with the desired temperature, involves a combinations of spurs of the derivatives of the desired solutions for the external regions (the last term). The solution of the problem (25)–(30) is described below.

Limiting Case of Zero Approximation. We now consider the limiting case of zero approximation where the last term on the left side of (27) can be disregarded as compared to the convective term. This is allowable if

$$\left| \frac{\chi}{2}\left(\left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right) \right| \bigg/ \left| U_{\text{ef}}\,(r,\,t)\,\frac{\partial T^{(0)}}{\partial r} \right| << 1\,, \tag{31}$$

i.e., $\chi a_{z1} R(\lambda_{z1} + \lambda_{z2})/(2Zu_0 h\lambda_{z1}) << 1$, where R and Z are the characteristic dimensions of the zone of temperature disturbances in the radial direction and in the z-axis direction, respectively. The estimations performed have shown that the left side of expression (31) corresponds to the 10^{-4}–10^{-3} range. Under these conditions, the limiting case of zero approximation can be used with a high degree of accuracy. The mathematical formulation of the problem will be written in the following form:

$$\frac{\partial T_1^{(0)}}{\partial t} - \frac{a_{r1}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T_1^{(0)}}{\partial r} \right) - \frac{\partial^2 T_1^{(0)}}{\partial z^2} = 0\,, \quad z > 1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{32}$$

$$\frac{\partial T_2^{(0)}}{\partial t} - \frac{a_{r2}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T_2^{(0)}}{\partial r} \right) - \frac{a_{z2}}{a_{z1}}\frac{\partial^2 T_2^{(0)}}{\partial z^2} = 0\,, \quad z < -1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{33}$$

$$\frac{\partial T^{(0)}}{\partial t} + U_{\text{ef}}\,(r,\,t)\left(\frac{\partial T^{(0)}}{\partial r} + \varepsilon_{\text{ef}}\,\frac{\partial P}{\partial r} \right) - \eta_{\text{ef}}\,\Pi\,\frac{\partial P}{\partial t} + q_1 - \frac{a_r}{a_{z1} r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T^{(0)}}{\partial r} \right) = 0\,,$$

$$|\,z\,| < 1\,, \quad r \geq 0\,, \quad t > 0\,; \tag{34}$$

$$T_1^{(0)}\,|_{z=1} = T_2^{(0)}\,|_{z=-1} = T^{(0)}\,; \tag{35}$$

$$T^{(0)}\,|_{t=0} = T_1^{(0)}\,|_{t=0} = T_2^{(0)}\,|_{t=0} = 0\,; \tag{36}$$

$$\lim T_i^{(0)}\big|_{r+|z|\to\infty} = 0\ . \tag{37}$$

In this system, the equations are not interrelated; therefore Eq. (34) can be solved independently of (32) and (33), and the solution $T^{(0)}(r,\ t)$ presented below is used as the boundary condition in solving problems on temperature fields in the first and second regions.

The corresponding problems are solved by the integral-transform method [12]. The resultant expressions have the form

$$T_1^{(0)}(r, z, t) = \int_0^t \frac{(z-1)\exp\left(-\dfrac{(z-1)^2}{4a_2\,(t-\theta)}\right)}{4\sqrt{\pi a_2^3 (t-\theta)^5}} \int_0^\infty x\exp\left(-\frac{r^2+x^2}{4a_2\,(t-\theta)}\right) T^{(0)}(x, \theta)\, I_0\left(-\frac{rx}{2a_2\,(t-\theta)}\right) dx\, d\theta,\ z > 1, \tag{38}$$

$$T_2^{(0)}(r, z, t) = \int_0^t \frac{(z+1)\exp\left(-\dfrac{(z+1)^2}{4a_1\,(t-\theta)}\right)}{4\sqrt{\pi a_1^3 (t-\theta)^5}} \int_0^\infty x\exp\left(-\frac{r^2+x^2}{4a_1\,(t-\theta)}\right) \times$$
$$\times\, T^{(0)}(x, \theta)\, I_0\left(-\frac{rx}{2a_1\,(t-\theta)}\right) dx\, d\theta\ ,\quad z < -1\ . \tag{39}$$

The first and higher approximations remedy the chief shortcoming of the zero approximation, which is that it does not describe the dependence of the temperature on the z coordinate in an interval of a pool.

Formulation of the Problem in the First Approximation. The problem is formulated in the first approximation with no regard for the radial heat conductivity on the assumption that the second terms of the order of ε in (10) and (11) and the third term of the order of ε^2 in Eq. (12) are equal to zero:

$$\frac{\partial T_1^{(1)}}{\partial t} - \frac{a_{r1}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1^{(1)}}{\partial r}\right) - \frac{\partial^2 T_1^{(1)}}{\partial z^2} = 0\ ,\quad z > 1\ ,\quad r \ge 0\ ,\quad t > 0\ ; \tag{40}$$

$$\frac{\partial T^{(1)}}{\partial t} + U_{\text{ef}}\frac{\partial T^{(1)}}{\partial r} - \frac{a_r}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(1)}}{\partial r}\right) - \chi\frac{\partial^2 T^{(2)}}{\partial z^2} = 0\ ,\quad |z| < 1\ ,\quad r \ge 0\ ,\quad t > 0\ ; \tag{41}$$

$$\frac{\partial T_2^{(1)}}{\partial t} - \frac{a_{r2}}{a_{z1} r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2^{(1)}}{\partial r}\right) - \frac{a_{z2}}{a_{z1}}\frac{\partial^2 T_2^{(1)}}{\partial z^2} = 0\ ,\quad z < -1\ ,\quad r \ge 0\ ,\quad t > 0\ ; \tag{42}$$

$$\left.\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=1} = \left.\frac{\partial T^{(2)}}{\partial z}\right|_{z=1}\ ,\qquad \left.\frac{\partial T_2^{(1)}}{\partial z}\right|_{z=-1} = \left.\frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial T^{(2)}}{\partial z}\right|_{z=-1}\ ; \tag{43}$$

$$T_1^{(1)}\big|_{z=1} = T^{(1)}\big|_{z=1}\ ,\quad T_2^{(1)}\big|_{z=-1} = T^{(1)}\big|_{z=-1}\ ; \tag{44}$$

$$T^{(1)}\big|_{t=0} = T_1^{(1)}\big|_{t=0} = T_2^{(1)}\big|_{t=0} = 0\ ; \tag{45}$$

$$\lim T_{1,2}^{(1)}\big|_{r+|z|\to\infty} = 0\ . \tag{46}$$

Let us "unlink" Eq. (41) involving expansion coefficients of the first and second order; to do this, we represent it as

$$\hat{L}T^{(1)} = \frac{\partial^2 T^{(2)}}{\partial z^2}\ ,\quad \hat{L} = \frac{1}{\chi}\left[\frac{\partial}{\partial t} + U_{\text{ef}}\frac{\partial}{\partial r} - \frac{a_r}{a_{z1}r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]. \tag{47}$$

Acting by the operator $\hat{L}$ upon expression (20), we obtain, according to (47),

$$\hat{L}T^{(1)} = \frac{z^2}{2}\hat{L}A\,(r,t) + z\hat{L}B\,(r,t) + \hat{L}E\,(r,t) = \frac{\partial^2 T^{(2)}}{\partial z^2}\ , \tag{48}$$

where $E = E(r,\ t)$ is independent of z. Equation (48) is equivalent to (41); here, $A(r,\ t)$ and $B(r,\ t)$ are sought from the zero approximation according to (24) and $E(r,\ t)$ is unknown. Having found an analytical expression of $E(r,\ t)$, we will construct a solution of the problem in the first approximation. To find $\hat{L}E(r,\ t)$, we will integrate expression (48) over z (M and N are integration constants):

$$\frac{\partial T^{(2)}}{\partial z} = \frac{z^3}{6}\hat{L}A\,(r,t) + \frac{z^2}{2}\hat{L}B\,(r,t) + z\hat{L}E\,(r,t) + M\,(r,t)\ , \tag{49}$$

$$T^{(2)} = \frac{z^4}{24}\hat{L}A\,(r,t) + \frac{z^3}{6}\hat{L}B\,(r,t) + \frac{z^2}{2}\hat{L}E\,(r,t) + zM\,(r,t) + N\,(r,t)\ . \tag{50}$$

Using (43) and (49), we construct a system of equations at boundaries $z = 1$ and $z = -1$:

$$\left.\frac{\partial T^{(2)}}{\partial z}\right|_{z=1} = \frac{1}{6}\hat{L}A\,(r,t) + \frac{1}{2}\hat{L}B\,(r,t) + \hat{L}E\,(r,t) + M\,(r,t) = \left.\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=1}\ , \tag{51}$$

$$\left.\frac{\partial T^{(2)}}{\partial z}\right|_{z=-1} = -\frac{1}{6}\hat{L}A\,(r,t) + \frac{1}{2}\hat{L}B\,(r,t) - \hat{L}E\,(r,t) + M\,(r,t) = \left.\frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=-1}\ . \tag{52}$$

The system of equations (51)–(52) is used for determining the unknowns $\hat{L}E(r,\ t)$ and $M(r,\ t)$:

$$\begin{aligned}\hat{L}E\,(r,t) &= \frac{1}{2}\left.\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=1} - \frac{1}{2}\frac{\lambda_{z2}}{\lambda_{z1}}\left.\frac{\partial T_2^{(1)}}{\partial z}\right|_{z=-1} - \frac{1}{6}\hat{L}A\,(r,t)\ ,\\ M\,(r,t) &= \frac{1}{2}\left.\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=1} + \frac{1}{2}\frac{\lambda_{z2}}{\lambda_{z1}}\left.\frac{\partial T_2^{(1)}}{\partial z}\right|_{z=-1} - \frac{1}{2}\hat{L}B\,(r,t)\ .\end{aligned} \tag{53}$$

Substituting the expressions for $A(r,\ t)$ and $B(r,\ t)$ from (24) and the expression for $\hat{L}E(r,\ t)$ from (53) into (48), we obtain a mathematical formulation of the problem for the first expansion coefficients:

$$\frac{\partial T_1^{(1)}}{\partial t} - \frac{a_{r1}}{a_{z1}r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1^{(1)}}{\partial r}\right) - \frac{\partial^2 T_1^{(1)}}{\partial z^2} = 0\ ,\quad z > 1\ ,\quad r \ge 0\ ,\quad t > 0\ ; \tag{54}$$

$$\frac{\partial T^{(1)}}{\partial t} + U_{\text{ef}} \frac{\partial T^{(1)}}{\partial r} - \frac{a_r}{a_{z1} r} \frac{\partial}{\partial r}\left(r \frac{\partial T^{(1)}}{\partial r} \right) - \frac{\chi}{2}\left(\left.\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left.\frac{\partial T_2^{(1)}}{\partial z}\right|_{z=-1} \right) = \chi \left(\frac{z^2}{4} + \frac{z}{2} - \frac{1}{12} \right) \times$$

$$\times \hat{L}\left(\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1} \right) - \chi \frac{\lambda_{z2}}{\lambda_{z1}} \left(\frac{z^2}{4} - \frac{z}{2} - \frac{1}{12} \right) \hat{L}\left(\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=-1} \right), \quad z < -1 , \quad r \geq 0 , \quad t > 0 ; \tag{55}$$

$$\frac{\partial T_2^{(1)}}{\partial t} - \frac{a_{r2}}{a_{z1} r} \frac{\partial}{\partial r}\left(r \frac{\partial T_2^{(1)}}{\partial r} \right) - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(1)}}{\partial z^2} = 0 , \quad z < -1 , \quad r \geq 0 , \quad t > 0 ; \tag{56}$$

$$T_1^{(1)} |_{z=1} = T^{(1)} |_{z=1} , \quad T_2^{(1)} |_{z=-1} = T^{(1)} |_{z=-1} ; \tag{57}$$

$$T^{(1)} |_{t=0} = T_1^{(1)} |_{t=0} = T_2^{(1)} |_{t=0} = 0 ; \tag{58}$$

$$\lim T_{1,2}^{(1)} |_{r+|z| \to \infty} = 0 . \tag{59}$$

To solve this problem, it is necessary to have an additional condition. This condition has been obtained by averaging Eq. (3) over the interval of the pool from $z = -1$ to $z = 1$.

Deduction of the Additional Integral Condition. Let us successively average each term of Eq. (3) over the interval of the pool $-1 < z < 1$. According to the boundary conditions (4) and (5), we have

$$\frac{\chi}{2\varepsilon} \int_{-1}^{1} \frac{\partial^2 T}{\partial z^2} dz = \frac{\chi}{2\varepsilon} \left.\frac{\partial T}{\partial z}\right|_{z=-1}^{z=1} = \frac{\chi}{2\varepsilon} \left(\left.\frac{\partial T}{\partial z}\right|_{z=1} - \left.\frac{\partial T}{\partial z}\right|_{z=-1} \right) = \frac{\chi}{2} \left(\left.\frac{\partial T_1}{\partial z}\right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left.\frac{\partial T_2}{\partial z}\right|_{z=-1} \right). \tag{60}$$

We write the final formulation of the problem for determining the average temperature $\langle T \rangle$ in the interval of the pool and the corresponding temperatures in the top and in the base of the pool:

$$\frac{\partial T_1}{\partial t} - \frac{a_{r1}}{a_{z1} r} \frac{\partial}{\partial r}\left(r \frac{\partial T_1}{\partial r} \right) - \frac{\partial^2 T_1}{\partial z^2} = 0 , \quad z > 1 , \quad r \geq 0 , \quad t > 0 ; \tag{61}$$

$$\frac{\partial T_2}{\partial t} - \frac{a_{r2}}{a_{z1} r} \frac{\partial}{\partial r}\left(r \frac{\partial T_2}{\partial r} \right) - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2}{\partial z^2} = 0 , \quad z < -1 , \quad r \geq 0 , \quad t > 0 ; \tag{62}$$

$$\frac{\partial \langle T \rangle}{\partial t} + U_{\text{ef}}(r, t) \frac{\partial \langle T \rangle}{\partial r} - \frac{a_r}{a_{z1} r} \frac{\partial}{\partial r}\left(r \frac{\partial \langle T \rangle}{\partial r} \right) - f(r, t) = \frac{\chi}{2}\left(\left.\frac{\partial T_1}{\partial z}\right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left.\frac{\partial T_2}{\partial z}\right|_{z=-1} \right), \ |z| < 1, r \geq 0, t > 0; \tag{63}$$

$$T_1 |_{z=1} = T_2 |_{z=-1} = \langle T \rangle ; \tag{64}$$

$$\langle T \rangle |_{t=0} = T_1 |_{t=0} = T_2 |_{t=0} = 0 ; \tag{65}$$

$$\lim T_{1,2}\big|_{r+|z|\to\infty} = 0 . \tag{66}$$

It is easy to verify that the problem on determination of the average temperature in a pool obtained in this way coincides with the problem in the zero approximation (25)–(30). It follows from the uniqueness of solution of the corresponding problems that $\langle T\rangle = T^{(0)}$; therefore, averaging series (8) gives

$$\langle T\rangle = T^{(0)} + \varepsilon\,\langle T^{(1)}\rangle + \varepsilon^2\,\langle T^{(2)}\rangle + ... + \varepsilon^n\,\langle T^{(n)}\rangle + ... ;\quad \varepsilon\,\langle T^{(1)}\rangle + \varepsilon^2\,\langle T^{(2)}\rangle + ... + \varepsilon^n\,\langle T^{(n)}\rangle + ... = 0 . \tag{67}$$

From (67) follows the additional condition imposed on the solutions of the first, second, and higher approximations:

$$\langle T^{(i)}\rangle\big|_{r=r_1} = 0 ,\quad i = 1, 2 ,... . \tag{68}$$

In (68), averaging is performed over any cylindrical surface $r = r_1$. It has been established that the best approximation to the desired solution is obtained in the case where r_1 coincides with the radius of the surface, on which the boundary conditions are set.

In view of (68), problem (54)–(59) has a unique solution, which allows one to calculate the corrections to the zero approximation. The zero and first approximations are quite sufficient for the majority of practical calculations.

Equality (68) makes it possible to determine the physical meaning of the zero approximation. It ascertains that the solution in the zero approximation corresponds to the determination of the temperature averaged over the thickness of the pool. Hence it also follows that, when $\varepsilon \to 0$, the limit of the solution corresponds to the averaging of the desired solution over a z interval of the pool. Moreover, it follows from the physical considerations that the vertical heat-conductivity component increases ($\lambda_z \to \infty$) when ε tends to zero. Thus, $T^{(0)}$ can be formally considered as a limit of the desired solution at an infinite, vertical heat-transfer conductivity component λ_z. To put it otherwise, when $\lambda_z \to \infty$, the limit of the desired solution corresponds to the averaging along the z coordinate over the pool interval $|z| < 1$.

Since, in problem (61)–(66), averaging was performed over the z interval of the pool, clearly the zero approximation is independent of the z coordinate. The first approximation $T^{(1)}$ allows one to determine the relation between the temperature in a pool interval and the z coordinate. However, there must be no changes in the average temperatures when this relation is taken into account; therefore, the first approximation satisfies the condition of equality of its average values to zero (68).

The Temperature Field in an Oil-Gas Pool in the Limiting Case of Zero Approximation. To calculate temperature fields by the relations obtained, it is necessary to solve hydrodynamic problems for calculating the rate of convective heat transfer, the effective Joule–Thomson coefficient, the effective adiabatic coefficient, and Π:

$$\mathbf{U}_{\text{ef}} = \sum_{i=1}^{3} \mathbf{v}_i c_i \rho_i / c_{\text{p}} , \tag{69}$$

$$\varepsilon_{\text{ef}} = \left[\left(\sum_{i=1}^{3} \varepsilon_i \mathbf{v}_i c_i \rho_i\right)\left(\sum_{i=1}^{3} \mathbf{v}_i c_i \rho_i\right)\right] \Big/ \left(\sum_{i=1}^{3} \mathbf{v}_i c_i \rho_i\right)^2 , \tag{70}$$

$$\eta_{\text{ef}} = \sum_{i=1}^{3} \rho_i s_i c_i \eta_i \Big/ \left(\sum_{i=1}^{3} \rho_i s_i c_i\right) ,\quad \Pi = m \sum_{i=1}^{3} s_i \rho_i c_i / c_{\text{p}} . \tag{71}$$

The mathematical formulation of the problem is described by the mass and momentum-balance equations, the equation of free-gas state, and the Henry law. In the case where the rate of filtration of the carrying phase is equal to that of the gas dissolved in the liquid $\mathbf{v} = \mathbf{v}_2$, the mass-conservation equation has the form

$$\frac{\partial (ms_1\rho_1)}{\partial t} + \nabla (\rho_1 \mathbf{v}_1) = 0 , \quad \frac{\partial (ms_2\rho_2)}{\partial t} + \nabla (\rho_2 \mathbf{v}_1) = - q , \quad \frac{\partial (ms_3\rho_3)}{\partial t} + \nabla (\rho_3 \mathbf{v}_3) = q ,$$

$$s_1 = s_2 = 1 - s , \quad s_1 + s_3 = 1 . \tag{72}$$

The lower indices 1, 2, and 3 are related to the carrying phase, the dissolved gas, and the free gas, respectively. For the filtration model, we use the Darcy law, which for the carrying phase and the free gas ($s_3 = s$), has the form

$$\mathbf{v}_1 = - \frac{k}{\mu_1} f_1 (s) \nabla P , \quad \mathbf{v}_3 = - \frac{k}{\mu_3} f_3 (s) \nabla P . \tag{73}$$

We use the Van der Waals equation of free-gas state

$$\left(P + \frac{b_1 \rho_3^2}{M_0^2} \right) \left(\frac{M_0}{\rho_3} - b_2 \right) = R_0 \tilde{T} . \tag{74}$$

According to the Henry law [13], the concentration of the dissolved gas is proportional to the pressure P:

$$\frac{\rho_2}{\rho_2 + \rho_1} = \alpha P . \tag{75}$$

Let us assume that the liquid phase is incompressible and the porosity is constant. Then, combining the last two equations of (72) and substituting the expressions for the filtration rates (73) into them, we obtain following system of equations:

$$\frac{\partial s}{\partial t} + \frac{k}{\mu_1 m} \nabla [f_1 (s) \nabla P] = 0 , \tag{76}$$

$$\frac{\partial}{\partial t} \Big[(1 - s) \rho_2 + s \rho_3 \Big] - \frac{k}{\mu_1 m} \nabla \left[\left(f_1 (s) \rho_2 + f_3 (s) \frac{\mu_1}{\mu_3} \rho_3 \right) \nabla P \right] = 0 . \tag{77}$$

For quasiequilibrium processes, the saturation s is a function of the pressure and temperature $s(P, T)$. Since, in the problem considered, the relative pressure differences are larger by some factors than the relative changes in temperature, it is advantageous to use the barotropic approximation $s = s(P)$, $\rho_3 = \rho_3(P)$. In this case, the system of equations (76)–(77) takes the form

$$\frac{ds}{dP} \frac{\partial P}{\partial t} + \frac{k}{\mu_1 m} f_1 (s) \Delta P + \frac{k}{\mu_1 m} \frac{df_1 (s)}{dP} (\nabla P)^2 = 0 , \tag{78}$$

$$\frac{d}{dP} \Big[(1 - s) \rho_2 + s \rho_3 \Big] \frac{\partial P}{\partial t} - \frac{k}{\mu_1 m} \left[\left(f_1 (s) \rho_2 + f_3 (s) \frac{\mu_1}{\mu_3} \rho_3 \right) \right] \nabla P -$$

$$- \frac{k}{\mu_1 m} \frac{d}{dP} \left[\left(f_1 (s) \rho_2 + f_3 (s) \frac{\mu_1}{\mu_3} \rho_3 \right) \right] (\nabla P)^2 = 0 . \tag{79}$$

The solutions of Eqs. (78) and (79) will coincide if the physical quantity — pressure — has a unique value, which is possible on condition that the ratios between the corresponding coefficients of these equations are equal:

$$\frac{f_1(s)\,\rho_2 + f_3(s)\,\rho_3\,\dfrac{\mu_1}{\mu_3}}{f_1(s)} = \frac{\dfrac{d}{dP}\left[f_1(s)\,\rho_2 + f_3(s)\,\dfrac{\mu_1}{\mu_3}\,\rho_3\right]}{\dfrac{d}{dP}\,f_1(s)} = -\frac{\dfrac{d}{dP}\,[(1-s)\,\rho_2 + s\rho_3]}{\dfrac{ds}{dP}} = c\,(P)\,. \tag{80}$$

Having determined $c(P)$ from the condition for the saturation pressure $\rho_2(P = P_s) = \rho_{20} = \rho_1 \alpha P_s/(1 - \alpha P_s)$, we find

$$\frac{f_3(s)}{f_1(s)} = \frac{\mu_3}{\mu_1}\,\frac{\rho_{20} - \rho_2}{\rho_3} = \frac{\rho_1\mu_3}{\rho_3\mu_1}\left(\frac{1}{1 - \alpha P_s} - \frac{1}{1 - \alpha P}\right) = \xi_1 \xi_2\,, \tag{81}$$

$$\xi_1 = \frac{\rho_1\mu_3}{\rho_3\mu_1}\,,\ \xi_2 = \frac{1}{1 - \alpha P_s} - \frac{1}{1 - \alpha P}\,,\ s = \frac{\rho_{20} - \rho_2}{\rho_3 + \rho_{20} - \rho_2} = \frac{f_3(s)\,\mu_1/(f_1(s)\,\mu_3)}{1 + f_3(s)\,\mu_1/(f_1(s)\,\mu_3)} \quad \text{or} \quad \frac{f_3(s)\,\mu_1}{f_1(s)\,\mu_3} = \frac{s}{1 - s}\,; \tag{82}$$

$$s = (1 + \xi_2^{-1}\xi_3)^{-1}\,,\quad \xi_3 = \rho_3/\rho_1\,. \tag{83}$$

It should be noted that relations (81)–(83) result from the continuity equation, i.e., from the mass-conservation law, and clearly the requirement for the uniqueness of the pressure field. Substituting relations (82) or (83) into (76) or (77), we obtain an equation for the pressure P in the filtration of a liquid:

$$\frac{d}{dP}\left[\left(1 + \frac{\rho_3}{\rho_{20} - \rho_2}\right)^{-1}\right]\frac{\partial P}{\partial t} + \frac{k}{\mu_1 m}\,\nabla\left\{f_1\left[\left(1 + \frac{\rho_3}{\rho_{20} - \rho_2}\right)^{-1}\right]\nabla P\right\} = 0\,.$$

In the particular case, applicable to sufficient accuracy to practical calculations, the corresponding pressure fields can be considered as stationary, since the temperature fields are established much more slowly as compared to the pressure fields. An analytical solution of the hydrodynamic problem has been constructed under these conditions [14]. It is also assumed that, at the initial instant of time, the pressure in the well declines instantaneously to P_w and then remains constant, $P = P_w$ ($r = r_0$, $t > 0$). The condition $P = P_c$ ($r = R_c$, $t > 0$) is set at the distant boundary of the porous medium.

Relations (81)–(83) allow one to construct a solution of the hydrodynamic problem in general form for the functions $f_1(s)$ or $f_3(s)$, which are determined experimentally or on the basis of solution of additional experimental problems.

For a porous medium representing a system of cylindrical tubes of the same diameter, in which gas bubbles alternate realistically with liquid bubbles, the expression for the phase permeability of the liquid has the form

$$f_1(s(P)) = \begin{cases} (1 + \xi_1\xi_2)^{-1}\,, & P < P_s\,; \\ 1\,, & P > P_s\,. \end{cases} \tag{84}$$

The rate of filtration of the liquid in the zones with free gas and in the zones where free gas is absent is described as

$$\mathbf{v}_1 = \begin{cases} -k\left(P_c - P_s - \displaystyle\int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)\Big/\,\mu_1 r \ln(R_c/r_0)\,, & P_w < P_s\,; \\ -\dfrac{k}{\mu_1}\,\dfrac{P_c - P_w}{\ln(R_c/r_0)}\,\dfrac{1}{r}\,, & P_s \le P_w < P_c\,, \end{cases} \tag{85}$$

where $\xi' = (1 - \alpha P_s)^{-1} - (1 - \alpha P')^{-1}$.

The expressions obtained for the phase permeability and the rate of filtration of the liquid with a dissolved gas allow one to construct an expression for the pressure-gradient distribution:

$$\frac{dP}{dr} = \begin{cases} (1+\xi_1\xi_2)\left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right) \Big/ r \ln \frac{R_c}{r_0}, & P < P_s ; \\ \frac{P_c - P_s}{\ln \frac{R_c}{r_s}} \frac{1}{r} = \left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right) \Big/ r \ln \frac{R_c}{r_0}, & P > P_s . \end{cases} \tag{86}$$

The dependence of the pressure P on the radial coordinate r is determined by the inverse function

$$r = r_s \exp\left[\ln \frac{R_c}{r_0} \int\limits_{P_s}^{P} \frac{dP'}{1+\xi_1\xi'} \Big/ \left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right)\right], \quad r < r_s , \quad P < P_s , \quad P_w < P_s ; \tag{87}$$

$$r = r_s \exp\left[(P - P_s) \ln \frac{R_c}{r_0} \Big/ \left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right)\right], \quad r > r_s , \quad P > P_s , \quad P_w < P_s . \tag{88}$$

The radius r_s of the zone from which gas outflow begins is determined by the expression

$$r_s = R_c \exp\left[(P - P_s) \ln \frac{r_0}{R_c} \Big/ \left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right)\right], \quad P_w < P_s . \tag{89}$$

Solving the hydrodynamic problem, we determined the density fields of the free-gas sources:

$$q = \begin{cases} \frac{\rho_1 \alpha k (1+\xi_1\xi_2)}{r^2 (1-\alpha P)^2 \mu_1 \ln^2 (R_c/r_0)} \left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right)^2, & P < P_s . \\ 0 , & P > P_s . \end{cases} \tag{90}$$

Taking into account the solution of the hydrodynamic problem and the Henry law, we obtain an expression for the effective Joule–Thomson coefficient:

$$\varepsilon_{ef} = \begin{cases} \frac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_4 + \varepsilon_3 c_3 \xi_2}{c_1 + c_2\xi_4 + c_3\xi_2}, & P < P_s ; \\ \frac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_5}{c_1 + c_2 \xi_5}, & P > P_s , \end{cases} \tag{91}$$

where $\xi_4 = \alpha P/(1 - \alpha P)$ and $\xi_5 = \alpha P_s/(1 - \alpha P_s)$.

Formula (91) shows the dependence of the effective Joule–Thomson coefficient on the pressure, and its relation to the radius is specified parametrically by joining the expression for the pressure distribution, obtained from (86) in implicit form, to (91):

$$\int\limits_{P_s}^{P} \frac{dP'}{1+\xi_1\xi'} \left(P_c - P_s - \int\limits_{P_s}^{P_w} \frac{dP'}{1+\xi_1\xi'}\right) \ln \frac{r}{r_s} \Big/ \ln \frac{R_c}{r_0}, \quad r < r_s , \quad P < P_s , \quad P_w < P_s ; \tag{92}$$

$$P = P_s + \left(P_c - P_s - \int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1 \xi'} \right) \ln \frac{r}{r_s} \Big/ \ln \frac{R_c}{r_0} \,, \quad r > r_s \,, \quad P > P_s \,, \quad P_w < P_s \,. \tag{93}$$

In the case of filtration of a degassed oil in the zone of degassing and in the zone where liberation of gas from the carrying phase is absent, the effective adiabatic coefficient has the form

$$\eta_{ef} = \begin{cases} \dfrac{c_1 \eta_1 + \xi_4 c_2 \eta_2 + \xi_2 c_3 \eta_3}{c_1 + \xi_4 c_2 + \xi_2 c_3} \,, & P < P_s \,; \\ \dfrac{c_1 \eta_1 + \xi_5 c_2 \eta_2}{c_1 + \xi_5 c_2} \,, & P > P_s \,. \end{cases} \tag{94}$$

At pressures larger or smaller than the saturation pressure, the effective rate of convective heat transfer is written as

$$U_{ef} = \begin{cases} \dfrac{\rho_1 v_1 \,(c_1 + c_2 \xi_4 + c_3 \xi_2)\,(\xi_2 + \xi_3)}{m\,(\rho_1 c_1 \xi_3 + \rho_1 c_2 \xi_3 \xi_4 + \rho_3 c_3 \xi_2) + (1 - m)\,\rho_0 c_0\,(\xi_2 + \xi_3)} \,, & P < P_s \,; \\ \dfrac{\rho_1 v_1 \,(c_1 + c_2 \xi_5)}{m \rho_1 \,(c_1 + c_2 \xi_5) + (1 - m)\,\rho_0 c_0} \,, & P > P_s \,. \end{cases} \tag{95}$$

In the case of filtration of a gassed oil, the heat capacity per unit volume is determined by the following expression:

$$c_p = \begin{cases} \dfrac{m \xi_3 \rho_1}{\xi_2 + \xi_3}\,(c_1 + c_2 \xi_4) + \dfrac{m \rho_3 c_3 \xi_2}{\xi_2 + \xi_3} + (1 - m)\,\rho_0 c_0 \,, & P < P_s \,; \\ m \rho_1\,(c_1 + c_2 \xi_5) + (1 - m)\,\rho_0 c_0 \,, & P > P_s \,. \end{cases} \tag{96}$$

Below is given a solution of the problem in the limiting case of zero approximation where the heat transfer to the environment is disregarded. It corresponds to the theory of barothermal effect with allowance for the phase transition caused by the degassing. We used the corrected equation of the barothermal effect

$$\frac{\partial T}{\partial t} + U_{ef}\,(P) \left(\frac{\partial T}{\partial r} + \varepsilon_a\,(P) \frac{\partial P}{\partial r} \right) - \eta_a\,(P)\,\Pi\,\frac{\partial P}{\partial t} = 0 \,, \tag{97}$$

where ε_a and η_a are the apparent Joule–Thomson and adiabatic coefficients:

$$\varepsilon_a = \frac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_4 + \varepsilon_3 c_3 \xi_2 - L\alpha/(1 - \alpha P)^2}{c_1 + c_2 \xi_4 + c_3 \xi_2} \,, \quad \eta_a = \frac{\eta_1 c_1 + \eta_2 c_2 \xi_4 + \eta_3 c_3 \xi_2 + L\alpha/(1 - \alpha P)^2}{c_1 + c_2 \xi_4 + c_3 \xi_2} \,. \tag{98}$$

The quantities U_{ef}, ε_a, η_a, $\partial P/\partial r$, Π, and $\partial P/\partial t$ are functions of the pressure P and the radial coordinate r, which, according to (87) and (88), is also a function of the pressure P. Hence it follows that the desired solution of Eq. (97) can be represented as a function of the pressure P, which is taken as an independent variable. Thus, we will assume that the temperature T is related to the pressure P and the time t. Let us transform Eq. (97) from variables r and t to the variables P and t. To do this, we will use the equality $\frac{\partial T}{\partial r} = \frac{\partial T}{\partial P} \frac{\partial P}{\partial r}$:

$$\frac{\partial T}{\partial t} + U\,(P) \frac{\partial T}{\partial P} = F_1\,(P) + F_2\,(P) \,, \tag{99}$$

where

$$U(P) = U_{ef}\frac{\partial P}{\partial r}\,; \quad F_1(P) = -\,\varepsilon_a U_{ef}\frac{\partial P}{\partial r}\,; \quad F_2(P) = \eta_a \Pi \frac{\partial P}{\partial t}\,.$$

The final formulation of the problem includes the boundary and initial conditions. Below we consider two cases of initial conditions:

1) the initial temperature is independent of the pressure P

$$T\big|_{t=0} = 0\,; \tag{100}$$

2) the initial temperature is a function of the radial coordinate r, which is reduced, using dependences (87) and (88), to the initial condition

$$T\big|_{t=0} = T_0(P)\,. \tag{101}$$

Equation (99) is solved by the method of characteristics, i.e., by a change from the Euler to the Lagrange variables. The equation for the characteristics has the form

$$\frac{dP}{dt} = U(P)\,. \tag{102}$$

Integrating (102) over the pressure from P_1 to P (left side) and over the time from t_1 to t (right side), we obtain

$$\int\limits_{P_1}^{P} \frac{dP'}{U(P')} = \int\limits_{t_1}^{t} d\tau\,. \tag{103}$$

Having designated the primitive in interval (103) as $\zeta(P)$, we find

$$\zeta(P) - \zeta(P_1) = t - t_1\,. \tag{104}$$

A solution of Eq. (104) is the expression

$$P = \zeta^{-1}(t - t_1 + \zeta(P_1))\,. \tag{105}$$

Along the characteristics, the equation for the temperature has the form

$$\frac{dT}{dt} = F_1\left[\zeta^{-1}(t - t_1 + \zeta(P_1))\right] + F_2\left[\zeta^{-1}(t - t_1 + \zeta(P_1))\right]. \tag{106}$$

Integrating (106) with the zero initial condition (100) and joining (104), we obtain a solution of Eq. (99):

$$T = \int\limits_0^t \left\{F_1\left[\zeta^{-1}(\tau - t_1 + \zeta(P_1))\right] + F_2\left[\zeta^{-1}(\tau - t_1 + \zeta(P_1))\right]\right\} d\tau\,, \quad \zeta(P) - \zeta(P_1) = t - t_1\,. \tag{107}$$

Having eliminated P_1 and t_1 from (107), we construct a solution of the nonstationary problem on the barothermal effect in the filtration of a liquid with a dissolved gas:

$$T = \int\limits_0^t \left\{F_1\left[\zeta^{-1}(\tau - t_1 + \zeta(P))\right] + F_2\left[\zeta^{-1}(\tau - t_1 + \zeta(P))\right]\right\} d\tau\,. \tag{108}$$

In a similar manner, we have obtained a solution of Eq. (99) with the initial condition (101):

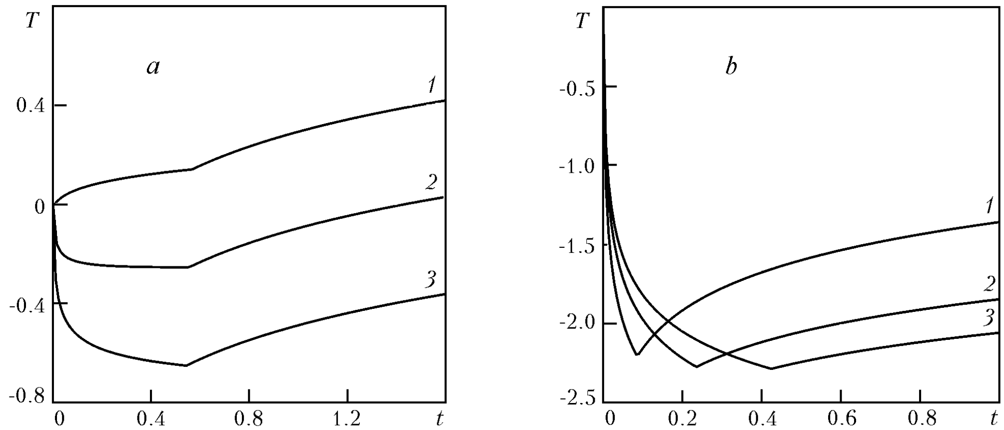

Fig. 2. Time dependence of the barothermal effect for different values of the gas solubility (a): 1) $\alpha = 10^{-9}$; 2) $4 \cdot 10^{-9}$; 3) $7 \cdot 10^{-9}$ 1/Pa, and different radius of the power-supply circuit (b): 1) $R_c = 20$; 2) 50; 3) 100 m. T, K; t, h.

$$T = \int_0^t \left\{ F_1 \left[\zeta^{-1} (\tau - t_1 + \zeta (P)) \right] + F_2 \left[\zeta^{-1} (\tau - t_1 + \zeta (P)) \right] \right\} d\tau + T_0 \left[\zeta^{-1} (\zeta (P) - t) \right] . \tag{109}$$

The solutions (108) or (109) can also be used as a zero approximation for calculating the temperature in the surrounding rocks in the limiting case of zero approximation (38) and (39).

In the case of stationary filtration where the adiabatic effect is ignored, the energy equation for a stable temperature $\partial T / \partial t = 0$ is obtained from (99):

$$\frac{dT}{dP} + \varepsilon_a (P) = 0 .$$

A solution of the stationary problem on the barothermal effect in the filtration of a liquid with a dissolved gas has the form

$$T = \begin{cases} \dfrac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_5}{c_1 + c_2 \xi_5} (P_c - P) , \quad P > P_s , \quad r_s < r < R_c ; \\ \varepsilon_{ef} (P_c - P_s) + \displaystyle\int_{P_s}^{P} \frac{L\alpha - (\alpha P')^2 [\varepsilon_1 c_1 + \varepsilon_2 c_2 \alpha P' / (1 - \alpha P') + \varepsilon_3 c_3 \xi']}{(1 - \alpha P')^2 [c_1 + c_2 \alpha P' / (1 - \alpha P') + c_3 \xi']} dP' , \\ P_w < P_s , \quad r_0 < r < r_s . \end{cases} \tag{110}$$

Figure 2a shows the time dependence of the barothermal effect in the filtration of the gassed liquid. It is seen from the figure that at $\alpha = 10^{-9}$ 1/Pa (curve 1) the temperature effect is positive and the pool is heated with time. At $\alpha = 4 \cdot 10^{-9}$ 1/Pa (curve 2), the pool is cooled for the time $t < 0.58$ h and is then heated, and for the large times $t > 1.4$ h the temperature effect takes positive values. At $\alpha = 7 \cdot 10^{-9}$ 1/Pa (curve 3), the pool is cooled for the time $t < 0.58$ h, and then the temperature begins to increase during a period of time $0.58 < t < 1.6$ h but remains in the range of negative values at all times.

Figure 2b shows the time dependences of the barothermal effect for different values of the power-supply-circuit radius. The calculations were performed for the gas-solubility coefficient $\alpha = 10^{-8}$ 1/Pa.

Figure 3 shows the dependences of the barothermal effect on the distance from the center of the well and on the pressure. The calculations were performed in accordance with dependence (110) for the oil with dissolved methane at $M = 0.016$ kg/mole at the following pressures and parameters of the pool: $P_c = 2 \cdot 10^7$ Pa, $P_w = 10^7$ Pa, $P_s = 1.4 \cdot 10^7$ Pa, $k = 10^{-12}$ m^2, and $m = 0.2$.

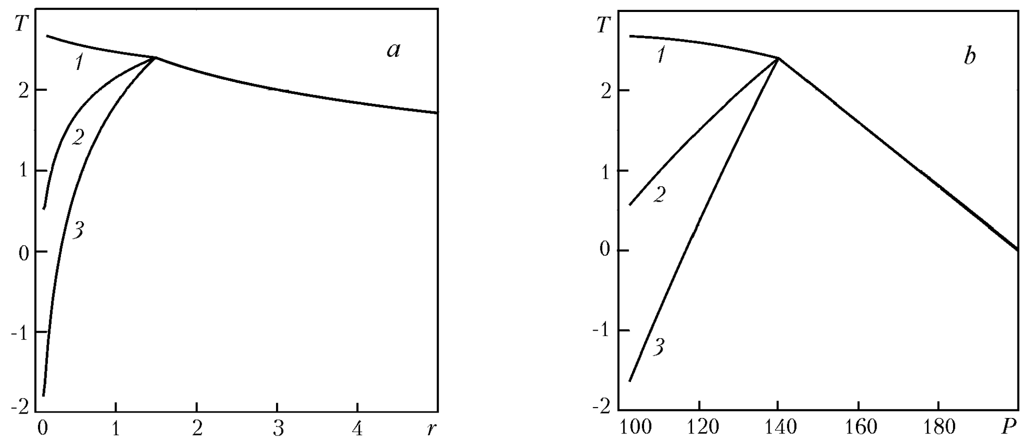

Fig. 3. Dependence of the temperate field on the distance from the center of the well (a) and on the pressure (b) for different values of the gas solubility: 1) $\alpha = 4{\cdot}10^{-9}$; 2) $7{\cdot}10^{-9}$; 3) 10^{-8} 1/Pa. T, K; r, m; P, atm.

The calculations correspond to a liquid movement from the pool to the well. In the case where $\alpha = 4{\cdot}10^{-9}$ 1/Pa (curve 1), the temperature of the pool in the zone where degassing is absent, $r_s < r < R_c$, increases along the liquid-movement path, and then, at $r = r_s$, curve 1 has a deflection. Curve 1 increases with decrease of the radius in the degassing zone. Curve 2 has been constructed for the gas solubility $\alpha = 7{\cdot}10^{-9}$ 1/Pa. Since a monophase oil moves in the zone where degassing is absent, curves 1, 2, and 3 coincide because the change in the temperature in this zone is due to the Joule–Thomson effect in the liquid. In the degassing zone $r_0 < r < r_s$, curve 2 decreases with decrease in the radius but remains in the range of positive values. Curve 3 has been constructed for the gas solubility $\alpha = 10^{-8}$ 1/Pa. The runs of curves 2 and 3 are similar; the only difference is that, at $r = r_0$, the change in the temperature of the pool takes negative values for the third curve. This allows the conclusion that, as the gas solubility increases, heating of the liquid flowing from the pool decreases and is changed to cooling.

Solution of the Problem on the Zero Approximation. Let us write problem (25)–(30) in the zero approximation, ignoring additionally, for simplicity, the radial heat conductivity for the first and second regions. This is possible in the case where

$$a_{r1}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right)\Big/ a_{z1}\frac{\partial^2 T_1}{\partial z^2} << 1 \quad \text{or} \quad \frac{a_{r1}Z^2}{a_{z1}R^2} << 1\,,$$

where R and Z are the characteristic zones of temperature disturbances in the radial direction and in the z-axis direction, respectively. Since the convective heat transfer is dominant in the first case and the conductive heat transfer is dominant in the second case, we have $Z \sim \sqrt{at}$ and $R \sim Ut$ for estimation. Substituting $U \sim 10^{-4}$ m/sec, $a \sim 10^{-7}$ m^2/sec, and $t \sim 10^5$ sec and taking into account that $a_{z1} \sim a_{r1}$, we obtain $a_{r1}Z^2/(a_{z1}R^2) \sim 10^{-4} << 1$. This estimation shows that the radial heat conductivity can be ignored.

Taking into account the foregoing, we write the mathematical formulation of the problem in the zero approximation:

$$\frac{\partial T_1^{(0)}}{\partial t} - \frac{\partial^2 T_1^{(0)}}{\partial z^2} = 0\,, \quad z > 1\,, \quad r \ge 0\,, \quad t > 0\,; \tag{111}$$

$$\frac{\partial T_2^{(0)}}{\partial t} - \frac{a_{z2}}{a_{z1}}\frac{\partial^2 T_2^{(0)}}{\partial z^2} = 0\,, \quad z < 1\,, \quad r \ge 0\,, \quad t > 0\,; \tag{112}$$

$$\frac{\partial T^{(0)}}{\partial t} + U_{\text{ef}}(r)\frac{\partial T^{(0)}}{\partial r} - \frac{\chi}{2}\left(\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}}\left.\frac{\partial T_2^{(0)}}{\partial z}\right|_{z=-1}\right) = f(r, t), \quad |z| < 1, \quad r \geq 0, \quad t > 0; \tag{113}$$

$$T_1^{(0)}|_{z=1} = T_2^{(0)}|_{z=-1} = T^{(0)}; \tag{114}$$

$$T^{(0)}|_{t=0} = T_1^{(0)}|_{t=0} = T_2^{(0)}|_{t=0} = 0; \tag{115}$$

$$\lim T_{1,2}^{(0)}|_{r+|z|\to\infty} = 0. \tag{116}$$

In accordance with [14, 15], it is assumed that the rate of convective heat transfer is independent of time:

$$U_{\text{ef}} = U_{\text{ef}}(r) = -\frac{J}{2r}.$$

Using the Laplace–Carson transform

$$T_i^{(0)\text{im}} = p\int_0^\infty \exp(-pt)\, T_i^{(0)}(t)\, dt, \tag{117}$$

we change the problem (111)–(116) to the image space

$$pT_1^{(0)\text{im}} - \frac{\partial^2 T_1^{(0)\text{im}}}{\partial z^2} = 0, \tag{118}$$

$$pT_2^{(0)\text{im}} - \frac{\lambda_{z2}}{\lambda_{z1}}\frac{\partial^2 T_1^{(0)\text{im}}}{\partial z^2} = 0, \tag{119}$$

$$pT^{(0)\text{im}} + U_{\text{ef}}(r)\frac{\partial T^{(0)\text{im}}}{\partial r} - \frac{\chi}{2}\left(\left.\frac{\partial T_1^{(0)\text{im}}}{\partial z}\right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}}\left.\frac{\partial T_2^{(0)\text{im}}}{\partial z}\right|_{z=-1}\right) = f(r, p), \tag{120}$$

$$T_1^{(0)\text{im}}|_{z=1} = T_2^{(0)\text{im}}|_{z=-1} = T^{(0)\text{im}}, \tag{121}$$

$$\lim T_{1,2}^{(0)\text{im}}|_{r+|z|\to\infty} = 0. \tag{122}$$

Let us write the solutions of Eqs. (118) and (119), taking into account (121), for the regions $z > 1$ and $z < -1$:

$$T_1^{(0)\text{im}} = T^{(0)\text{im}} \exp\left[-\sqrt{p}\,(z-1)\right], \quad T_2^{(0)\text{im}} = T^{(0)\text{im}} \exp\left[\sqrt{p\frac{a_{z1}}{a_{z2}}}\,(z+1)\right]. \tag{123}$$

In the image space, expressions (24) can be represented in the form

$$A^{\text{im}} = -\frac{\sqrt{p}}{2}\left(1 + \frac{\lambda_{z2}}{\lambda_{z1}}\sqrt{\frac{a_{z1}}{a_{z2}}}\right)T^{(0)\text{im}} = -\sqrt{p}\,\Lambda_1 T^{(0)\text{im}}\,, \quad B^{\text{im}} = -\frac{\sqrt{p}}{2}\left(1 - \frac{\lambda_{z2}}{\lambda_{z1}}\sqrt{\frac{a_{z1}}{a_{z2}}}\right)T^{(0)\text{im}} = -\sqrt{p}\,\Lambda_2 T^{(0)\text{im}}\,, \tag{124}$$

where

$$\Lambda_1 = \frac{1}{2}\left(1 + \frac{\lambda_{z2}}{\lambda_{z1}}\sqrt{\frac{a_{z1}}{a_{z2}}}\right); \quad \Lambda_2 = \frac{1}{2}\left(1 - \frac{\lambda_{z2}}{\lambda_{z1}}\sqrt{\frac{a_{z1}}{a_{z2}}}\right). \tag{125}$$

Taking into account (124), for the equations of barothermal effect in images (120) we obtain

$$(p + \chi\sqrt{p}\,\Lambda_1)\,T^{(0)\text{im}} - \frac{J}{2r}\frac{dT^{(0)\text{im}}}{dr} = f(r, p)\,. \tag{126}$$

Equation (126) is solved by the method of separation of variables:

$$T^{(0)\text{im}} = \frac{2}{J}\int_0^\infty f(r', p)\exp\left(-\frac{p + \chi\sqrt{p}\,\Lambda_1}{J}\left(r'^2 - r^2\right)\right)r'dr'\,. \tag{127}$$

Using (127), it is easy to construct a solution in images for the first and second regions:

$$T_1^{(0)\text{im}} = \frac{2}{J}\int_0^\infty f(r', p)\exp\left(-\frac{p + \chi\sqrt{p}\,\Lambda_1}{J}\left(r'^2 - r^2\right) - \sqrt{p}\,(z-1)\right)r'dr'\,; \tag{128}$$

$$T_2^{(0)\text{im}} = \frac{2}{J}\int_0^\infty f(r', p)\exp\left(-\frac{p + \chi\sqrt{p}\,\Lambda_1}{J}\left(r'^2 - r^2\right) + \sqrt{p\,\frac{a_{z1}}{a_{z2}}}\,(z+1)\right)r'dr'\,. \tag{129}$$

A changeover to the space of originals is made with the use of the known operation relations

$$\exp(-\gamma p)\, f(p) \to \varphi(t - \gamma)\, I(t - \gamma)\,, \tag{130}$$

$$\exp(-\gamma p)\exp(\beta\sqrt{p}) \to \operatorname{erfc}\left(\frac{\beta}{2\sqrt{t-\gamma}}\right)I(t-\gamma)\,, \quad I(x) = \begin{cases} 1\,, & x \ge 0\,; \\ 0\,, & x < 0\,, \end{cases} \tag{131}$$

where $\beta = \chi\Lambda_1\alpha$ and $\gamma = (r'^2 - r^2)/J$.

For each of the three regions, the resultant solution of the problem in the zero approximation has the form

$$T^{(0)} = \frac{2}{J}\int_r^\infty \frac{d}{dt}\int_0^t f(r', t-\tau)\operatorname{erfc}\left(\frac{\chi\Lambda_1(r'^2 - r^2)}{2J\sqrt{\tau - (r'^2 - r^2)J^{-1}}}\right)I(\tau - (r'^2 - r^2)J^{-1})\,d\tau r'dr',\ |z| < 1,\ t > 0; \tag{132}$$

$$T_1^{(0)} = \frac{2}{J}\int_r^\infty \frac{d}{dt}\int_0^t f(r', t-\tau)\operatorname{erfc}\left(\frac{\dfrac{\chi\Lambda_1(r'^2 - r^2)}{J} + z - 1}{2\sqrt{\tau - (r'^2 - r^2)J^{-1}}}\right)I(\tau - (r'^2 - r^2)J^{-1})\,d\tau r'dr'\,, \quad z > 1\,, \quad t > 0\,; \tag{133}$$

$$T_2^{(0)} = \frac{2}{J}\int_r^{\infty}\frac{d}{dt}\int_0^t f(r', t-\tau)\ \mathrm{erfc}\left(\frac{\dfrac{\chi\Lambda_1(r'^2 - r^2)}{J} + \sqrt{\dfrac{a_{z1}}{a_{z2}}}\ (z+1)}{2\sqrt{\tau - (r'^2 - r^2)J^{-1}}}\right) I(\tau - (r'^2 - r^2)J^{-1})\ d\tau r' dr',\ z < -1,\ t > 0. \quad (134)$$

These solutions are the desired formulas for calculating the fields of barothermal effect in a pool and in the surrounding rocks. The solutions for higher approximations are constructed in a similar manner.

Construction of a Solution in the First Approximation. Ignoring the radial heat conductivity and taking into account with allowance for the additional condition (68), we write The final formulation of the problem (54)–(59) for the first coefficient of the expansion in terms of ε as

$$\frac{\partial T_1^{(1)}}{\partial t} - \frac{\partial^2 T_1^{(1)}}{\partial z^2} = 0\ ,\quad z > 1\ ,\quad t > 0\ ; \quad (135)$$

$$\frac{\partial T^{(1)}}{\partial t} + U_{\mathrm{ef}}\frac{\partial T^{(1)}}{\partial r} - \frac{\chi}{2}\left.\frac{\partial T_1^{(1)}}{\partial z}\right|_{z=1} + \frac{\chi}{2}\frac{\lambda_{z2}}{\lambda_{z1}}\left.\frac{\partial T_2^{(1)}}{\partial z}\right|_{z=-1} = \chi\left(\frac{z^2}{4} + \frac{z}{2} - \frac{1}{12}\right)\hat{L}\left(\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=1}\right) -$$

$$- \chi\left(\frac{z^2}{4} - \frac{z}{2} - \frac{1}{12}\right)\hat{L}\left(\frac{\lambda_{z2}}{\lambda_{z1}}\left.\frac{\partial T_1^{(0)}}{\partial z}\right|_{z=-1}\right),\quad |z| < 1\ ,\quad r \geq 0\ ,\quad t > 0\ ; \quad (136)$$

$$\frac{\partial T_2^{(1)}}{\partial t} - \frac{a_{z2}}{a_{z1}}\frac{\partial^2 T_2^{(1)}}{\partial z^2} = 0\ ,\quad z < -1\ ,\quad t > 0\ ; \quad (137)$$

$$T_1^{(1)}\big|_{z=1} = T^{(1)}\big|_{z=1},\quad T_2^{(1)}\big|_{z=-1} = T^{(1)}\big|_{z=-1}\ ; \quad (138)$$

$$T^{(1)}\big|_{t=0} = T_1^{(1)}\big|_{t=0} = T_2^{(1)}\big|_{t=0} = 0\ ; \quad (139)$$

$$\lim T_{1,2}^{(1)}\big|_{r+|z|\to\infty} = 0\ ; \quad (140)$$

$$\int_{-1}^{1} T^{(1)}\ (z, r = r_1)\ dz = 0\ . \quad (141)$$

A solution of problem (135)–(141) has been obtained on the basis of the Laplace–Carson transform. On changeover to the space of originals with the use of the inverse-transformation formulas [16], the expressions for the first coefficient of the expansion in terms of ε take the form

$$T^{(1)} = -\frac{2}{J\sqrt{\pi}}\left[\frac{\Lambda_1}{2}\left(z^2 - \frac{1}{3}\right) + \Lambda_2 z\right]\int_r^{\infty} r' dr' \left\{\frac{d}{dt}\int_0^t \frac{f(r', t-\tau)}{(\tau - (r'^2 - r^2)\ J^{-1})^{1/2}} \times\right.$$

$$\left.\times \exp\left[-\frac{(r'^2 - r^2)^2\ \chi^2\Lambda_1^2}{4J^2\ (\tau - (r'^2 - r^2)\ J^{-1})}\right] I\ (\tau - (r'^2 - r^2)\ J^{-1})\ d\tau\right\} +$$

$$+\frac{2\chi\xi}{J^3\sqrt{\pi}}\int_r^{r_1} y\,dy\int_\infty^y r'dr'\left\{\frac{d}{dt}\int_0^t f(r', t-\tau)\,\frac{(r'^2-r^2)\,\chi\Lambda_1}{(\tau-(r'^2-r^2)\,J^{-1})^{3/2}}\times\right.$$

$$\left.\times\exp\left(-\frac{(r'^2-r^2)^2\,\chi^2\Lambda_1^2}{4J^2\,(\tau-(r'^2-r^2)\,J^{-1})}\right)I\,(\tau-(r'^2-r^2)\,J^{-1})\,d\tau\right\}, \tag{142}$$

$$T_1^{(1)}=-\frac{2}{J\sqrt{\pi}}\left(\frac{\Lambda_1}{3}+\Lambda_2\right)\int_r^\infty r'dr'\left\{\frac{d}{dt}\int_0^t\frac{f(r', t-\tau)}{(\tau-(r'^2-r^2)\,J^{-1})^{1/2}}\times\right.$$

$$\times\exp\left[-\frac{((r'^2-r^2)\,\chi\Lambda_1 J^{-1}+z-1)^2}{4\,(\tau-(r'^2-r^2)\,J^{-1})}\right]I\,(\tau-(r'^2-r^2)\,J^{-1})\,d\tau\Bigg\}+$$

$$+\frac{2\chi\xi}{J^2\sqrt{\pi}}\int_r^{r_1} y\,dy\int_\infty^y r'dr'\left\{\frac{d}{dt}\int_0^t f(r', t-\tau)\,\frac{(r'^2-r^2)\,\chi\Lambda_1 J^{-1}+z-1}{(\tau-(r'^2-r^2)\,J^{-1})^{3/2}}\times\right.$$

$$\left.\times\exp\left(-\frac{((r'^2-r^2)\,\chi\Lambda_1 J^{-1}+z-1)^2}{4\,(\tau-(r'^2-r^2)\,J^{-1})}\right]I\,(\tau-(r'^2-r^2)\,J^{-1})\,d\tau\right\}, \tag{143}$$

$$T_2^{(1)}=-\frac{2}{J\sqrt{\pi}}\left(\frac{\Lambda_1}{3}-\Lambda_2\right)\int_r^\infty r'dr'\left\{\frac{d}{dt}\int_0^t\frac{f(r', t-\tau)}{(\tau-(r'^2-r^2)\,J^{-1})^{1/2}}\times\right.$$

$$\times\exp\left[-\frac{((r'^2-r^2)\,\chi\Lambda_1 J^{-1}-\sqrt{a_{z1}/a_{z2}}\,(z+1))^2}{4\,(\tau-(r'^2-r^2)\,J^{-1})}\right]I\,(\tau-(r'^2-r^2)\,J^{-1})\,d\tau\Bigg\}+$$

$$+\frac{2\chi\xi}{J^2\sqrt{\pi}}\int_r^{r_1} y\,dy\int_\infty^y r'dr'\left\{\frac{d}{dt}\int_0^t f(r', t-\tau)\,\frac{(r'^2-r^2)\,\chi\Lambda_1 J^{-1}-\sqrt{a_{z1}/a_{z2}}\,(z+1)}{(\tau-(r'^2-r^2)\,J^{-1})^{3/2}}\times\right.$$

$$\left.\times\exp\left(-\frac{((r'^2-r^2)\,\chi\Lambda_1 J^{-1}-\sqrt{a_{z1}/a_{z2}}\,(z+1))^2}{4\,(\tau-(r'^2-r^2)\,J^{-1})}\right]I\,(\tau-(r'^2-r^2)\,J^{-1})\,d\tau\right\}, \tag{144}$$

where $\xi = \Lambda_2^2+\Lambda_1^2/3$. The final solution in the first approximation has the form

$$T=T^{(0)}+\frac{\lambda_{z1}}{\lambda_z}\,T^{(1)},\quad T_1=T_1^{(0)}+\frac{\lambda_{z1}}{\lambda_z}\,T_1^{(1)},\quad T_2=T_2^{(0)}+\frac{\lambda_{z1}}{\lambda_z}\,T_2^{(1)}. \tag{145}$$

We have performed calculations and constructed plots of the temperature distribution in the first approximation. Figure 4 shows the calculated dependences of the temperature T on the vertical coordinate z for different values of the parameters entering into the solution. Curve 1 corresponds to the solution in the zero approximation, curve 2

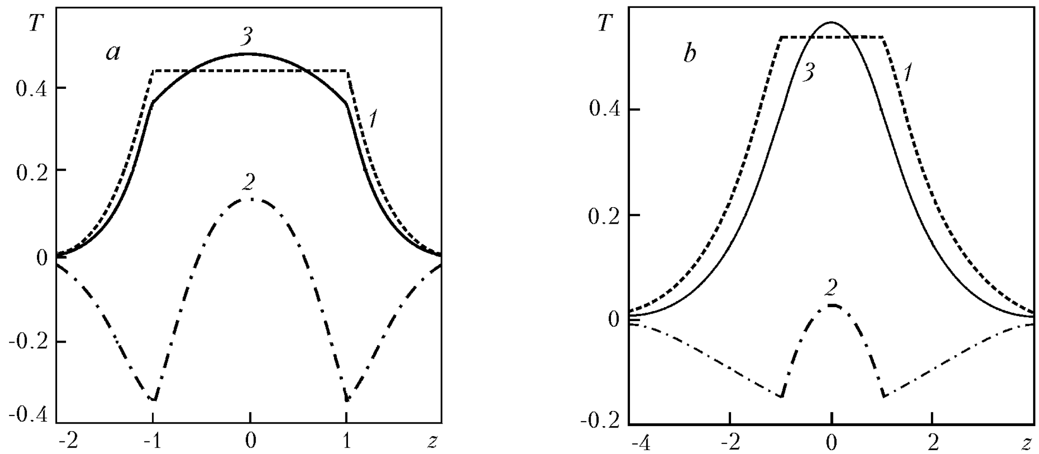

Fig. 4. Dependences of the temperature T on the z coordinate in the dimensionless coordinates h = 1 m, R_c = 50 m: a) t = 0.1, ε = 0.25; b) t = 1, ε = 1.

corresponds to the first expansion coefficient, and curve 3 corresponds to the first approximation. The calculations were performed on the assumption that $r = r_0$ and $r_1 = R_c$.

The calculations allow one to estimate the vertical dimensions of the zone in which the barothermal effect causes temperature changes. For example, for the dimensionless time t = 0.1 (see Fig. 4), the dimension of the zone of barothermal effect is 1 m and increases with time.

Curve 1 obtained for the zero approximation shows that in the pool interval $-1 < z < 1$ the temperature is constant as it must be in accordance with the "lumped capacitance scheme." The first expansion coefficient within the pool (curve 2) takes both negative and positive values. The solution in the first approximation (curve 3), in which the correction is taken into account, more actually reflects the temperature distribution in the pool since it depends on z. From Fig. 4 it is seen that, for small periods of time, the zero approximation describes the temperature distribution in the central region with a deficiency and at the edges of the pool with an excess. The zero approximation always gives excessive values of the temperature in the surrounding media.

When Fig. 4a is compared with Fig. 4b, it is apparent that the first expansion coefficient decreases in absolute value with time which also follows from the theoretical analysis of the expressions for the first and higher expansion coefficients. Such behavior of the coefficients makes it possible to attain a high accuracy of the calculations at $\varepsilon > 1$ for large times with the use of the first approximation.

Thus, the use of asymptotic methods allowed us to obtain an analytical solution of the problem on the temperature field of the barothermal effect in an oil-gas pool surrounded by the impenetrable rocks.

This work was carried out with financial support from the Russian Basic Research Foundation (grant 02-01-97908 2002AG).

NOTATION

A, B, E, F_i, M, N, auxiliary functions; a_p, thermal diffusivity of the saturated porous medium, m^2/sec; b_1, b_2, constants of the equation of state; c_i, heat capacity of the ith phase (component), J/(K·kg); c_p, heat capacity per unit volume of the saturated porous medium, J/(K·m^3; $f_i(s)$, phase permeability; H, total thickness of the pool; h, half-thickness of the pool, m; J, rate factor; k, permeability, m^2; L, heat of a phase transition, J/kg; $\hat{L}$, operator; m, porosity; M_0, molar mass, kg/mole; q, density function of the degassing sources (intensity of phase transitions), kg/(m^3·sec); q_1, density function of the heat sources; P_s, saturation pressure, Pa; P_c, pool pressure, Pa; P_w, pressure in the well, Pa; R_0, universal gas constant, J/(K·mole); R_c, radius of the power-supply circuit, m; r_0, radius of the well, m; r_s, radius of the saturation zone, m; r, z, cylindrical coordinates; s_i, saturation of the ith phase (component); $\tilde{T}$, absolute temperature, K; T, temperature anomaly (temperature difference), K; t, time, h; α, gas-solubility coefficient, 1/Pa; β, γ, constants; Δ, Laplace operator; ∇, Nabla operator; λ_i, heat conductivity coefficient, W/(m·K); ε, asymptotic expansion parameter; ε_i, Joule–Thomson coefficient of the *i*th phase (component), K/Pa; μ_i, viscosity of the *i*th phase (component), Pa·sec; ξ_i, auxiliary notations; η_i, adiabatic coefficient of the *i*th phase (component), K/Pa; ρ_i, density of the *i*th phase (component), kg/m^3; ρ_{20}, density of the dissolved gas at the saturation point, kg/m^3; τ, y, integration variables; $\mathbf{v}_i$, rate of filtration of the *i*th phase (component), m/sec; p, parameter of the Laplace–Carcson transform; $\text{erfc}\,(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-u^2)du$; $I_0(x)$, Bessel function of the imaginary argument; $I(t)$, unit Heaviside function. Subscripts: p, porous; i, number of the phase (component); c, contour; d, dimensional; ef, effective; s, gas saturation; w, well; r, z, directions; a, apparent; im, image. A prime in a designation corresponds to an integration variable or a function dependent on it.

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